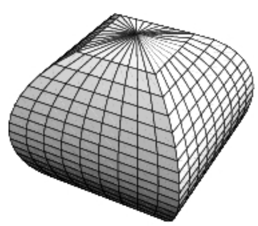
Worksheet 24, Math H53

Surfaces and Surface Integrals

Tuesday, April 30, 2013

1. Find a parametric representation for the plane through the origin that contains the vectors $\mathbf{i} - \mathbf{j}$ and $\mathbf{j} - \mathbf{k}$.
2. Find a parametric representation for the part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz-plane.
3. Find a parametric representation for the part of the cylinder $y^2 + z^2 = 16$ that lies between the planes $x = 0$ and $x = 5$.
4. Find an equation of the tangent plane to the parametric surface $x = u^2 + 1$, $y = v^3 + 1$, $z = u + v$ at the point $(5, 2, 3)$.
5. Find the area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.
6. Find the area of the part of the cone $z^2 = x^2 + y^2$ that lies between the plane $y = x$ and the cylinder $y = x^2$.
7. Find the area of the helicoid (or spiral ramp) with vector equation $\mathbf{r}(u, v) = \langle u\cos v, u\sin v, v\rangle$, $0 \leq u \leq 1$, $0 \leq v \leq \pi$.
8. Consider the Steinmetz Solid, which is the intersection of the two perpendicular cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 1$, pictured below. What is the surface area of this solid?



9. Evaluate the surface integral $\iint_X z\,dS$, where S is the surface $x = y + 2z^2$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.
10. Evaluate the surface integral $\iint_X y\,dS$, where S is the part of the paraboloid $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 4$.
11. Evaluate the surface integral $\iint_X \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = \langle x, -z, y\rangle$, where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant, with orientation toward the origin.
12. Let $\mathbf{F}$ be an inverse square field, that is $\mathbf{F}(\mathbf{r}) = c\mathbf{r}/\left|\mathbf{r}\right|^3$ for some constant c, where $\mathbf{r} = \langle x, y, z\rangle$. Show that the flux of $\mathbf{F}$ across a sphere S with center the origin is independent of the radius of S.
13. Suppose S is a surface of finite area and $\mathbf{F}(x, y, z) = \mathbf{C}$ is constant on S. Is the flux of $\mathbf{F}$ across S just $|\mathbf{C}|$ times the area of S?