Worksheet 20, Math H53
Vector Fields and Line Integrals

Thursday, April 11, 2013

1. Sketch the vector field $\mathbf{F}$ for the following vector-valued functions.

    (a) $\mathbf{F}(x,y) = 0.3\mathbf{i} - 0.4\mathbf{j}$

    (b) $\mathbf{F}(x,y) = y\mathbf{i} + (x+y)\mathbf{j}$

    (c) $\mathbf{F}(x,y) = \frac{y\mathbf{i}+x\mathbf{j}}{\sqrt{x^2+y^2}}$

    (d) $\mathbf{F}(x,y,z) = x\mathbf{k}$

2. Find the gradient vector field of f.

    (a) $f(x,y) = xe^{xy}$

    (b) $f(x,y) = \tan(3x-4y)$

3. Find the gradient vector field of f and sketch it.

    (a) $f(x,y) = \sqrt{x^2+y^2}$

    (b) $f(x,y) = x^2 - y$

4. Evaluate the line integral $\int_C x\sin y\, ds$, where C is the line segment from $(0,3)$ to $(4,6)$.

5. Evaluate the line integral $\int_C e^x\, dx$, where C is the arc of the curve $x = y^3$ from $(-1,-1)$ to $(1,1)$.

6. Evaluate the line integral $\int_C xyz^2\, ds$, where C is the line segment from $(-1,5,0)$ to $(1,6,4)$.

7. Evaluate the line integral $\int_C z^2\, dx + x^2\, dy + y^2\, dz$, where C is the line segment from $(1,0,0)$ to $(4,1,2)$.

8. Evaluate the line integral $\int_C \langle \sin x, \cos y, xz \rangle \cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t) = \langle t^3, -t^2, t \rangle$, $0 \le t \le 1$.

9. An object with mass m moves with position function $\mathbf{r}(t) = \langle a\sin t, b\cos t, ct \rangle$, $0 \le t \le \pi/2$. Find the work done on the object during this time period.

10. A 160-lb man carries a 25-lb can of paint up a helical staircase that encircles a silo of radius 20 ft. If the silo is 90 ft high and the man makes exactly three complete revolutions climbing to the top, how much work is done by the man against gravity?