Worksheet 17, Math H53
Multiple Integrals

Tuesday, April 2, 2013

1. Calculate the interated integral
$$\int_0^1 \int_0^3 e^{x+3y}\,dx\,dy$$

2. Calculate the double integral
$$\iint_R \frac{x}{1+xy}\,dA$$
over the region $R = [0,1] \times [0,1]$.

3. Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.

4. Evaluate the double integral
$$\iint_D y^2\,dA$$
over the triangular region D with vertices $(0,1)$, $(1,2)$, and $(4,1)$.

5. Sketch the region of integration and change the order of integration:
$$\int_0^2 \int_{x^2}^4 f(x,y)\,dy\,dx$$

6. Sketch the region of integration and change the order of integration:
$$\int_0^{\pi/2} \int_0^{\cos x} f(x,y)\,dy\,dx$$

7. Evaluate the integral by reversing the order of integration:
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$$

8. Use geometry, symmetry, or both to evaluate the double integral
$$\iint_D (2 + x^2y^3 - y^2 \sin x)\,dA,$$
where $D = \{(x,y) : |x| + |y| \leq 1\}$.

9. Find the Jacobian of the transformation: $x = u/v$, $y = v/w$, $z = w/u$.

10. Find the image of the square bounded by the lines $u = 0$, $u = 1$, $v = 0$, $v = 1$ under the transformation $x = v$, $y = u(1+v^2)$.

11. If $f(x,y)$ is a continuous function on the rectangular region $[a,b] \times [c,d]$ and
$$g(x,y) = \int_a^x \int_c^y f(s,t)\,dt\,ds$$
for $a < x < b$ and $c < y < d$, show that $g_{xy} = g_{yx} = f(x,y)$. What theorem from Chapter 14 does this result remind you of?

12. If f is a constant function, $f(x,y) = k$, and $R = [a,b] \times [c,d]$, show that

$$\iint_R k\, dA = k(b-a)(d-c).$$

13. Use the last result to show that

$$0 \leq \iint_R \sin(\pi x)\cos(\pi y)\, dA \leq \frac{1}{32}$$

where $R = \left[0, \frac{1}{4}\right] \times \left[\frac{1}{4}, \frac{1}{2}\right]$.