Worksheet 16, Math H53

Midterm II Review

Thursday, March 21, 2013

1. Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

2. Find parametric equations for the tangent line to the curve $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$ at the point $(1, 0, 1)$.

3. Find the length of the curve $\left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle$, where t varies between 0 and 1.

4. Find the limit if it exists, or show that it does not exist:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{yz}{x^2 + 4y^2 + 9z^2}$$

5. Show that the functinon f given by $f(\mathbf{x}) = |\mathbf{x}|$ is continuous on $\mathbb{R}^n$.

6. Verify that the conclusion of Clairaut's Theorem holds (that is, $u_{xy} = u_{yx}$) for $u = \ln(x + 2y)$.

7. Use show that the function $f(x, y) = x/(x+y)$ is differentiable at the point $(2, 1)$, and find a linearization of f at this point. Use this linearization to approximate $f(2.01, 1.1)$.

8. Let $z = \sin^{-1}(x - y)$, and let $x = s^2 + t^2$, $y = 1 - 2st$. Find $\partial z/\partial s$ and $\partial z/\partial t$.

9. Find the directional derivative of $f(x, y, z) = xe^y + ye^z + ze^x$ at the point $(0, 0, 0)$ in the direction of the vector $\mathbf{v} = \langle 5, 1, -2\rangle$. What is a general equation for the directional derivative of f at the point (a, b, c) in the direction of a unit vector $\mathbf{u} = \langle \alpha, \beta, \gamma\rangle$?

10. Find the local maximum and minimum values and saddle point of the function $f(x, y) = x^3 - 12xy + 8y^3$.

11. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the square $S = \{(x, y) |\, |x| \leq 1, |y| \leq 1\}$.

12. Find the absolute maximum and minimum values of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disk $D = \{(x, y)|x^2 + y^2 \leq 4\}$.

13. Among all the planes that are tangent to the surface $xy^2z^2 = 1$, find the ones that are farthest from the origin.