Worksheet 15, Math H53

More Lagrange Multipliers

Tuesday, March 19, 2013

1. Use Lagrange multipliers to find the minimum and maximum values of the function $f(x, y) = y^2 - x^2$ subject to the constraint $x^2/4 + y^2 = 1$.

2. Use Lagrange multipliers to find the minimum and maximum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$.

3. Use Lagrange multipliers to find the minimum and maximum values of the function $f(x, y, z, t) = x + y + z + t$ subject to the constraint $x^2 + y^2 + z^2 + t^2 = 1$.

4. Find the extreme values of $f(x, y, z) = yz + xy$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$.

5. Find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x - y = 1$ and $y^2 - z^2 = 1$.

6. Find the extreme values of $f(x, y) = e^{-xy}$ on the region described by the inequality $x^2 + 4y^2 \leq 1$.

7. Use Lagrange multipliers to find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.

8. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

9. (a) Maximize $\sum_{i=1}^{n} x_i y_i$ subject to the constraints $\sum_{i=1}^{n} x_i^2 = 1$ and $\sum_{i=1}^{n} y_i^2 = 1$.

   (b) Put

   $$x_i = \frac{a_i}{\sqrt{\sum a_j^2}} \quad \text{and} \quad y_i = \frac{b_i}{\sqrt{\sum b_j^2}}$$

   to show that

   $$\sum a_i b_i \leq \sqrt{\sum a_j^2}\sqrt{\sum b_j^2}$$

   for any numbers $a_1, \ldots, a_n, b_1, \ldots, b_n$. Have we seen this inequality before?

10. A rectangle with length L and width W is cut into four smaller rectangles by two lines parallel to the sides. Find the maximum and minimum values of the sum of the squares of the areas of the smaller rectangles.