Worksheet 13, Math H53
Maximal and Minimal Values

Tuesday, March 12, 2013

1. Find and identify the local maximum and minimum values and saddle points of the function $f(x, y) = (x - y)(1 - xy)$
2. A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of cardboard used.
3. Find and identify the local maximum and minimum values and saddle points of the function $f(x, y) = xy(1 - x - y)$
4. For functions of one variable it is impossible for a continuous function to have two local maxima and no local minimum. But for functions of two variables such functions exist. Show that the function
$$f(x, y) = -(x^2 - 1) - (x^2y - x - 1)^2$$
has only two critical points, but has local maxima at both of them. What characteristic of the graph of f allows the two local maxima to exist without any other critical points?
5. Find the absolute maximum and minimum values of the function $f(x, y) = 4x + 6y - x^2 - y^2$ on the domain $D = \{(x, y) : 0 \leq x \leq 4,\ 0 \leq y \leq 5\}$
6. Find three positive numbers whose sum is 100 and whose product is maximal.
7. Find and identify the local maximum and minimum values and saddle points of the function $f(x, y) = e^y(y^2 - x^2)$
8. Show that $f(x, y) = x^2+4y^2-4xy+2$ has an infinite number of critical points, and that the discriminant D is zero at each one. Then show that f has a local (and absolute) minimum at each critical point.
9. Find and identify the local maximum and minimum values and saddle points of the function $f(x, y) = e^x \cos y$
10. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?
11. Find the absolute maximum and minimum values of the function $f(x, y) = 2x^3 + y^4$ on the domain $D = \{(x, y) : x^2 + y^2 \leq 1\}$
12. Find an equation of the plane that passes through the point $(1, 2, 3)$ and cuts off the smallest volume in the first octant.