Worksheet 11, Math H53
More Partial Derivatives and Chain Rule

Tuesday, March 5, 2013

1. Find an equation of the tangent plane to the surface $z - xe^{xy} = 0$ at the point $(2, 0, 2)$.
2. Find an equation of the tangent plane to the surface $z - \ln(x - 2y) = 0$ at the point $(3, 1, 0)$.
3. Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + y/2$ at the point $(0, 0)$.
4. You are told that there is a function f whose partial derivatives are $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$. Should you believe it?
5. If $f(x, y) = x(x^2 + y^2)^{-3/2} e^{\sin(x^2 y)}$, find $f_x(1, 0)$. (There is an easy way and a hard way to do this–try to find the easy way!)
6. The paraboloid $x^2 + x + 2y^2 + z - 6 = 0$ intersects the plane $x = 1$ in a parabola. Find parametric equations for the tangent line to this parabola at the point $(1, 2, -4)$.
7. If $w = \ln \sqrt{x^2 + y^2 + z^2}$, with $x = \sin t$, $y = \cos t$ and $z = \tan t$, what is the value of dw/dt?
8. If $u = xe^{ty}$, and $x = \alpha^2\beta$, $y = \beta^2\gamma$, and $t = \gamma^2\alpha$, compute u_α, u_β and u_γ when $\alpha = -1$, $\beta = 2$, and $\gamma = 1$.
9. For what values of the number r is the function

$$f(x, y, z) = \begin{cases} \frac{(x+y+z)^r}{x^2+y^2+z^2} & \text{if } (x, y, z) \neq (0, 0, 0) \\ 0 & \text{if } (x, y, z) = (0, 0, 0) \end{cases}$$

   continuous?
10. If f is homogeneous of degree n, show that

$$x^2 f_{xx}(x, y) + xy f_{xy}(x, y) + yx f_{yx}(x, y) + y^2 f_{yy}(x, y) = n(n - 1) f(x, y)$$

11. Suppose f is a differentiable function of one variable. Show that all tangent planes to the surface $z = xf(y/x)$ intersect in a common point.
12. We have seen a formula for the derivative dy/dx of a function defined implicitly by an equation $F(x, y) = 0$, provided that F is differentiable and $F_y \neq 0$:

$$\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{F_x}{F_y}.$$

   Prove that if F has continuous second derivatives, then a formula for the second derivative of y is

$$\frac{d^2y}{dx^2} = -\frac{F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2}{F_y^3}.$$