Worksheet 9, Math H53

Differentiability, Linear Approximations, and Partial Derivatives

Tuesday, February 26, 2013

1. Find the first partial derivatives of the functions:
   - $f(p,q) = \tan^{-1}(pq^2)$
   - $F(\alpha,\beta) = \displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt$
   - $u = x^{y/z}$
   - $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$
2. Verify that the conclusion of Clairaut's Theorem holds for $u = \cos(x^2 y)$, that is, that $u_{xy} = u_{yx}$.
3. For $f(x,y) = \sin(2x+5y)$, find the value of f_{yxy}.
4. If $g(x,y,z) = \sqrt{1+xz} + \sqrt{1-xy}$, find g_{xyz}. Hint: Is there an easy way to compute this derivative?
5. If a,b,c are the sides of a triangle and A,B,C are the opposite angles, find $\partial A/\partial a$, $\partial A/\partial b$, $\partial A/\partial c$.
6. If $f(x,y) = \sqrt[3]{x^3+y^3}$, find $f_x(0,0)$.
7. Find an equation of the tangent plane to the surface $z = \sqrt{xy}$ at the point $(1,1,1)$.
8. Explain why the function $f(x,y) = 1 + x\ln(xy-5)$ is differentiable at the point $(2,3)$, and find the linearization $L(x,y)$ at this point.
9. Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$, and $f_y(2,5) = -1$, use a linear approximation to extimate $f(2.2, 4.9)$.
10. Find the differential of the function $T = \frac{v}{1+uvw}$.
11. Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.
12. Show that the function $f(x,y) = x^2+y^2$ is differentiable directly using the definition of differentiability.