Worksheet 6, Math H53

Midterm I Review

Thursday, February 14, 2013

1. Find a vector that has the same direction as $\langle -2, 4, 2\rangle$ but has length 6.

2. Find the scalar and vector projections of $\mathbf{a} = \langle -2, 3, -6\rangle$ onto $\mathbf{b} = \langle 5, -1, 4\rangle$.

3. For $\mathbf{a} = \langle 0, 1, 7\rangle$ and $\mathbf{b} = \langle 2, -1, 4\rangle$, find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

4. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 1, 2, 3\rangle$, $\mathbf{b} = \langle -1, 1, 2\rangle$, and $\mathbf{c} = \langle 2, 1, 4\rangle$.

5. If $\mathbf{c} = |\mathbf{a}|\,\mathbf{b} + |\mathbf{b}|\,\mathbf{a}$, where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are all nonzero vectors, show that $\mathbf{c}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$.

6. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, show that $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$.

7. Find equations of the plane:
   (a) Through the point $(2, 0, 1)$ and perpendicular to the line $x = 3t$, $y = 2 - t$, $z = 3 + 4t$.
   (b) Containing the line $x = 1+t$, $y = 2-t$, and $z = 4-3t$, which is parallel to the plane $5x+2y+z = 1$.
   (c) Through the points $(0, 1, 1)$, $(1, 0, 1)$, and $(1, 1, 0)$.
   (d) Passing through the points $(0, -2, 5)$ and $(-1, 3, 1)$ and is perpendicular to the plane $2z = 5x+4y$.

8. Find parametric equations and symmetric equations for the line through $(2, 1, 0)$ and perpendcular to both $\langle 1, 1, 0\rangle$ and $\langle 0, 1, 1\rangle$.

9. Reduce the equation
$$4y^2 + z^2 - x - 16y - 4z + 20 = 0$$
to one of the standard forms, classify the surface, and sketch it.

10. Identify the type of conic section whose equation is $y^2 = 16x^2 + 16$, and find the vertices and foci.

11. Find an equation for the ellipse with foci $(0, -1)$, $(8, -1)$, and with vertex $(9, -1)$.

12. Suppose $\mathbf{v}_1$ and $\mathbf{v}_2$ are vectors with $|\mathbf{v}_1| = 2$, $|\mathbf{v}_2| = 3$, and $\mathbf{v}_1 \cdot \mathbf{v}_2 = 5$. For each $n \geq 2$, let $v_{n+1} = \mathrm{proj}_{\mathbf{v}_{n-1}} \mathbf{v}_n$. Compute $\sum_{n=1}^{\infty} |\mathbf{v}_n|$.