Worksheet 5, Math H53

Quadratic Functions, Curves, Surfaces II

Tuesday, February 12, 2013

1. Find an equation for the parabola consisting of all points equidistant from the point $(-4, 0)$ and the line $x = 2$.

2. Find an equation of the ellipse with center $(-1, 4)$, and which has a vertex located at $(-1, 0)$, and a focus located at $(-1, 6)$.

3. Find an equation of the hyperbola with vertices $(-3, -4)$ and $(-3, 6)$, and foci $(-3, -7)$ and $(-3, 9)$.

4. Determine the type of curve represented by the equation

$$\frac{x^2}{k} + \frac{y^2}{k-16} = 1$$

for respectively $k > 16$, $0 < k < 16$, and $k < 0$. Show that the curves in the first two cases have the same foci, no matter what the value of k is.

5. Describe and sketch the surface $xy = 1$.

6. Reduce the equation

$$x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$$

to one of the standard forms, classify the surface, and sketch it.

7. Show that if an ellipse and a hyperbola have the same foci, then their tangent lines at each point of intersection are perpendicular.

8. Let $r > 1$, and consider all of the points which are r times the distance from the point $A = (-1, 0, 0)$ as from the point $B(1, 0, 0)$. Identify the surface, and describe its properties in terms of r.

9. Show that if the point (a, b, c) lies on the hyperbolic paraboloid $z = y^2 - x^2$, then the lines with parametric equations $x = a + t$, $y = b + t$, $z = c + 2(b - a)t$ and $x = a + t$, $y = b - t$, $z = c - 2(b + a)t$ both lie entirely on this paraboloid. This shows that the hyperbolic paraboloid is what is called a *ruled surface*; that is, it can be generated by the motion of a straight line. What other quadric surfaces are ruled surfaces?

10. Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.