

# Worksheet 1, Math H53

## Vectors and Geometry

Thursday, January 24, 2013

1. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
2. Prove that from segments congruent to the medians of a given triangle, another triangle can be formed.
3. Sides of one triangle are parallel to the medians of another. Prove that the medians of the latter triangle are parallel to the sides of the former one.
4. From medians of a given triangle, a new triangle is formed, and from its medians, yet another triangle is formed. Prove that the third triangle is similar to the first one, and find the coefficient of similarity.
5. Prove that if the sum of three unit vectors is equal to  $\mathbf{0}$ , then the angle between each pair of these vectors is equal to  $120^\circ$
6. For arbitrary vectors  $\mathbf{u}$  and  $\mathbf{v}$ , verify the equality:

$$|\mathbf{u} + \mathbf{v}|^2 + |\mathbf{u} - \mathbf{v}|^2 = 2|\mathbf{u}|^2 + 2|\mathbf{v}|^2,$$

and derive the theorem: the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of the sides.

7. Given a quadrilateral with perpendicular diagonals, show that every quadrilateral, whose sides are respectively congruent to the sides of the given one, has perpendicular diagonals.