

Math 55 Quiz 13  
November 21, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

**True or False.** Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. F The function  $G(x) = 1/(1-x)^2$  is the generating function for the sequence  $a_k = k$ .

b. F When discussing formal power series, convergence of the series as a function is of central importance.

c. T The function

$$G(x) = \frac{1}{1-x} \cdot \frac{1}{1-x^5}$$

is the generating function for the number of ways to make change for  $k$  dollars using \$1 and \$5 bills.



**Exercise.** Suppose we are given three non-disjoint sets  $A_1$ ,  $A_2$ , and  $A_3$ , and we select a random element out of their union. If each of the sets contains 100 elements, there are 50 common elements in each pair of sets, and there are 25 elements contained in all three sets, what is the probability of choosing an element which is in  $A_1$  or  $A_2$ ?

Since this is a uniform distribution, the probability we want is given by  $|A_1 \cup A_2| / |A_1 \cup A_2 \cup A_3|$ . We compute the sizes of these sets using inclusion-exclusion:

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 100 + 100 - 50 = 150$$

$$\begin{aligned} |A_1 \cup A_2 \cup A_3| &= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| \\ &\quad - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3| \\ &= 3 \cdot 100 - 3 \cdot 50 + 25 = 175. \end{aligned}$$

Thus the probability we want is:  $150/175 = 6/7$ .