Math 55 Quiz 13
November 21, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. F If $G(x)$ is the ordinary generating function of the sequence $a_0, a_1, a_2, \ldots$, then $x^2G(x)$ is the ordinary generating function of the sequence $\underset{x}{\underline{1, 1}}, a_0, a_1, a_2, \ldots$.

b. T The extended binomial theorem can be used to derive the binomial coefficient formula for the number of ways to distribute r indistinguishable biscuits to n distinguishable dogs.

c. T The principle of inclusion-exclusion can be used to count either the number of elements in a union of finite sets, or the number of elements in an intersection of finite sets.

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Exercise. What is the generating function for the sequence $\{c_k\}$, where c_k is the number of ways to make change for k dollars using \$1, \$5, \$10, and \$20 bills, assuming that we're short on small bills and only have three \$1 bills and two \$5 bills?

We will want to multiply together one generating function representing each type of bill. Since we have an unlimited number of \$10 and \$20 bills, we use an infinite geometric series for each of these denominations: $\frac{1}{1-x^{10}}$ and $\frac{1}{1-x^{20}}$.

For the small bills, since we have a limited supply we instead use a finite geometric series: $1+x+x^2+x^3$ and $1+x^5+x^{10}$. Thus the overall generating function is just

$$G(x) = (1+x+x^2+x^3)\cdot(1+x^5+x^{10})\cdot\frac{1}{1-x^{10}}\cdot\frac{1}{1-x^{20}}.$$