This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. __F__   The method of characteristic equations can be used to find a solution to the recurrence: $a_n = a_{n-1} + 2a_{n-2} + \cdots + (n-1)a_1$.

b. __T__   The Catalan number C_n describes the number of ways to parenthesize the product of $n + 1$ numbers to specify the order of multiplication.

c. __T__   The recurrence $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$ is linear, and has degree k.

❀

Exercise. Find a particular solution to the inhomogeneous recurrence relation $a_n = 2a_{n-1} + n$.

The inhomogeneous part of the recurrence is just $F(n) = n$, so there exists a particular solution of the form $a_n = An + B$ for some constants A and B. We will substitute this solution form into the recurrence to solve for these constants:

$$n = 0: \quad B = 2 \cdot (-A + B) + 0 \iff 2A - B = 0$$

$$n = 1: \quad A + B = 2 \cdot (B) + 1 \implies A - B = 1.$$

Solving, we get: $A = 1 + B$, $\quad 2(1 + B) - B = 2 + B = 0 \implies B = -2$

$\implies A = -1$, so a particular solution is $a_n = -n - 2$.