This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. __F__ Chebyshev's inequality:

$$p(|X - E(X)| \geq r) \leq V(X)/r^2$$

for X a random variable and $r > 0$, provides useful information for values of r <u>smaller than the standard deviation of X.</u>

b. __T__ If X and Y are independent random variables, then $E(XY) = E(X)E(Y)$.

c. __F__ If X is a random variable, a is a positive real number, and V denotes variance, then $V(aX) = aV(X)$.

$$a^2$$

✳

Exercise. Suppose that a list of n distinct integers contains the integer x at a random position. We want to find the position of x in the list using the following algorithm: At each step, we choose a random position in the list (possibly one we've already considered!) and check whether x is at that position. If it is, the algorithm terminates. If it isn't, the algorithm continues to search.

What is the expected number of steps needed to find the position of x in the list?

At each step, the process has a $1/n$ chance of terminating, and an $n-1/n$ chance of continuing. This is thus just a geometric distribution with success probability $p = 1/n$, so the expected value is given by $1/p = 1/(1/n) = n$.