Math 55 Quiz 11
November 9, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. F — Chebyshev's inequality gives an upper bound on how likely it is for a random variable to be equal to its expected value.

b. F — If X and Y are independent random variables and V denotes variance, then $V(X+Y) = V(X)+V(Y)$.

c. T — Suppose you have a biased coin which comes up heads with some probability p. If you flip this coin repeatedly until it comes up heads, a geometric distribution with parameter p describes the number of times you flipped the coin.

Exercise. In a simple game, a fair coin is flipped. If it comes up heads, you roll a six-sided die and get that number of points. If it comes up tails, you instead just get 2 points. What is the expected number of points you receive from this game?

Hint: You should rewrite this random variable as a combination of simpler random variables.

Let I_H be a random variable that is 1 if the coin flip is heads, and 0 if tails, and let $I_T = 1 - I_H$. Additionally let X be a random variable that takes the value of the die roll. Then the random variable for the number of points we receive is

$$P = I_H \cdot X + I_T \cdot 2$$

Since the coin flip is independent of the die roll, we know that $E(I_H \cdot X) = E(I_H) \cdot E(X)$, and we also know that the expected value of a roll of a 6-sided die is $7/2$, so

$$E(P) = E(I_H \cdot X + I_T \cdot 2) = E(I_H \cdot X) + E(I_T \cdot 2) = E(I_H)E(X) + 2 \cdot E(I_T)$$
$$= 1/2 \cdot 7/2 + 1/2 \cdot 2 = 11/4.$$