

Math 55 Quiz 7
October 12, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T An extended binary tree is either the empty set, or any tree formed by connecting two previously constructed extended binary trees to a new root vertex.

b. T The following

$$f(0) = 0, f(1) = 2, \quad f(n) = f(\lfloor n/3 \rfloor + 1) * f(n-1), n \geq 2$$

is a valid recursive definition.

c. T There are 60 one-to-one functions from a set with 3 elements to a set with 5 elements.

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Exercise. Recall that the Fibonacci numbers f_n are defined by the recursive definition:

$$f_0 = 0, \quad f_1 = 1, \quad f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 2$$

Prove that $f_1 + f_3 + \dots + f_{2n-1} = f_{2n}$ for any positive integer n .

We proceed by induction. For the base case, we have

$$1 = f_1 = f_2 = 1,$$

so the sum holds for $n=1$. Suppose now that the sum holds for $n=k$. Then we have

$$\begin{aligned} (f_1 + f_3 + \dots + f_{2k-1}) + f_{2k+1} &\stackrel{\text{I.H.}}{=} f_{2k} + f_{2k+1} \\ &= f_{2k+2} = f_{2(k+1)}. \end{aligned}$$

Thus the sum holds for $n=k+1$ as well. By mathematical induction, the sum holds for all n .