(b)

Math 55 Quiz 6
October 5, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T Mathematical induction can be used to prove the summation formula for geometric progressions.

b. F The well-ordering property states that every set of nonnegative integers has a least element.

c. T Mathematical induction can be applied with a base case which is a negative integer.

✤

Exercise. Use mathematical induction (not a direct argument) to prove that 2 divides $n^2 + n$ whenever n is a positive integer.

Let $P(n)$ denote the proposition that $n^2+n \equiv 0 \pmod 2$.

As a base case, note that $1^2+1 = 2 \equiv 0 \pmod 2$, so $P(1)$ holds. For the inductive step, let $k \geq 1$, and suppose $P(k)$ holds. Then

$$(k+1)^2+(k+1) = k^2+3k+2 = (k^2+k)+2k+2$$
$$\overset{\text{I.H.}}{\equiv} 0+2(k+1) \equiv 0 \pmod 2.$$

Thus in this case, $P(k+1)$ holds, and this completes the inductive step. By mathematical induction, we can conclude that $P(n)$ holds for all $n \geq 1$.