(a)

Math 55 Quiz 6

October 5, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T When using strong mathematical induction to prove a proposition $P(n)$ for all positive integers n, the inductive hypothesis when proving $P(k+1)$ is that $P(j)$ holds for $1 \leq j \leq k$.

b. F In certain circumstances, an inductive proof may still be valid with the base case omitted.

c. F The well-ordering property is equivalent to mathematical induction, but it cannot be used to prove the validity of strong mathematical induction.

✷

Exercise. Use mathematical induction to prove that, for each positive integer n,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=1-\frac{1}{2^n}.$$

Let $P(n)$ denote the proposition that $1/2+1/4+1/8+\cdots+1/2^n=1-1/2^n$.
For the base case, note that $1/2=1-1/2$, so $P(1)$ holds.
For the inductive step, let $k\geq 1$, and suppose $P(k)$ holds.
Then

$$\left(\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^k}\right)+\frac{1}{2^{k+1}} \overset{\text{I.H.}}{=} 1-1/2^k+1/2^{k+1} = 1+\frac{1-2}{2^{k+1}} = 1-1/2^{k+1}.$$

This proves $P(k+1)$, and completes the inductive step.
Thus by mathematical induction, $P(n)$ holds for all $n\geq 1$.