(b)

Math 55 Quiz 5
September 28, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. F Given the system of modular arithmetic equations

$$x \equiv 11 \pmod{12}$$
$$x \equiv 20 \pmod{25}$$

$(12, 25) = 1$

$12 \cdot 25 = 300 \neq 600$

the Chinese Remainder Theorem tells us that there exists a unique solution x to this system, modulo 600.

b. T Two integers a and b are relatively prime if their ~~greatest~~ least common multiple is $a \cdot b$.

c. F $(1110101)_2$ is the binary representation of the decimal number 125.

$1+4+16+32+64 = 117 \neq 125$

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Exercise. Use the Euclidean algorithm to find an inverse of 10 modulo 73.

Hint: An inverse of 10 modulo 73 is an integer x such that $10x \equiv 1 \pmod{73}$, i.e. such that $10x + 73y = 1$ for some integer y.

The Euclidean algorithm gives the following:

$73 = 7 \cdot 10 + 3$ $\quad 3 = 73 - 7 \cdot 10$

$10 = 3 \cdot 3 + 1$ $\quad \longrightarrow \quad 1 = 10 - 3 \cdot 3$

$3 = 3 \cdot 1 + 0$

Thus, $1 = 10 - 3 \cdot 3 = 10 - 3 \cdot (73 - 7 \cdot 10) = 22 \cdot 10 - 3 \cdot 73$.

This means that $x = 22$ is an inverse of 10 mod 73.