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Math 55 Quiz 5
September 28, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

- a. T 2 is a primitive root modulo 11. 2, 4, 8, 5, 10, 9, 7, 3, 6, 1
- b. T For any positive integers a and b , there exist integers x and y such that $ax + by = \gcd(a, b)$.
- c. F It is always possible to find a solution x of a linear congruence of the form $ax \equiv b \pmod{m}$, where $m > 1$ and $a, b \in \mathbb{Z}$, if we know that $\gcd(b, m) = 1$.

X



Exercise. What does Fermat's Little Theorem say about powers of 7 modulo 13? Use this fact to find the value of $7^{121} \pmod{13}$.

(If you forget what Fermat's Little Theorem says, try to simplify the expression anyway. ☺)

Fermat's Little Theorem tells us that $7^{12} \equiv 1 \pmod{13}$.

Thus we have that $7^{121} = 7 \cdot 7^{120} = 7 \cdot (7^{12})^{10}$
 $\equiv 7 \cdot 1^{10} \pmod{13}$,

so 7^{121} is equal to 7 mod 13.