Math 55 Quiz 3
September 14, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. T $\emptyset \subseteq \emptyset$.

b. T For a positive real number x, if x is irrational, then $\sqrt{x}$ is irrational.

c. F If A_d denotes the set $\{n \in \mathbb{Z} : n \text{ is divisible by } d\}$ of integers divisible by d, then $\bigcap_{d=1}^{\infty} A_d$ is the empty set.

Exercise. Prove that for any nonnegative integer n, there exists a nonnegative integer m such that $m^2 \leq n < (m+1)^2$.

Write $\sqrt{n} = a + \varepsilon$, where a is a nonnegative integer and $0 \leq \varepsilon < 1$. Then $0 \leq a \leq a + \varepsilon = \sqrt{n}$, so since everything is nonnegative, we can square the inequalities to get $a^2 \leq n$. Further, we have that $a+1 > a + \varepsilon = \sqrt{n} \geq 0$, so similarly we can square the inequalities to get $n < (a+1)^2$. Thus the choice of $m = a$ satisfies the desired properties.