

Math 55 Quiz 3
September 14, 2016

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

- a. F Given two positive numbers x and y , the arithmetic mean $(x + y)/2$ is less than or equal to the geometric mean \sqrt{xy} .
- b. F If x is a set element, then $x \in \mathcal{P}(\{x\})$, where \mathcal{P} denotes the power set.
- c. T If A and B are sets, then $A \cap B$ denotes the set of all x for which $x \in A$ and $x \in B$.

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Exercise. Prove that for any real number x , there is a rational number in the interval $(x, x + \frac{1}{\pi})$.

Since $\pi < 4$, we have that $4x + \frac{4}{\pi} > 4x + 1$, so in particular there is an integer in the interval $(4x, 4x + \frac{4}{\pi})$ since it has length greater than 1. Call this integer m . Then we have

$$4x < m < 4x + \frac{4}{\pi},$$

so

$$x < \frac{m}{4} < x + \frac{1}{\pi}.$$

Thus $m/4$ is a rational number in $(x, x + \frac{1}{\pi})$.