

Quiz 5

Math 54 Linear Algebra and DE with Professor Voiculescu

Tuesday, 6 March 2014

Name: \_\_\_\_\_

**Problem 1.** (10 points) Let  $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$ .

(a) (2 points) Determine the rank  $A$  and the  $\dim \text{Row } A$ .

(b) (8 points) Find the bases for  $\text{Col } A$  and  $\text{Row } A$ .

**Problem 2.** (4 points) The set  $\mathcal{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$  is a basis for  $\mathbb{P}_2$ . Find the coordinate vector of  $\mathbf{p}(t) = 1 + 3t - 6t^2$  relative to  $\mathcal{B}$ .

**Problem 3.** (6 points) True or False. Provide a justification or a counter-example.

(a)  $\mathbb{P}_2$  and  $\mathbb{R}_3$  are isomorphic, i.e. there exists an isomorphism between the two spaces.

(b) Let  $W$  be a vector space spanned by the set  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ . Then for all  $\mathbf{x} \in W$ , there exists a unique set of scalars  $c_1, c_2, c_3$  such that  $\mathbf{x} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + c_3\mathbf{b}_3$ .

(c) Let  $A$  be an  $m \times n$  matrix. Rank  $A^T$  + Nullity  $A = n$