Quiz 5 Solution
Math 54 Linear Algebra and DE with Professor Voiculescu
Tuesday, 6 March 2014

Problem 1. (*10 points*) Let $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$.

(a) (*2 points*) Determine the rank A and the dim Row A.

Solution: The echelon form of A is $\begin{bmatrix} 1 & -4 & 9 & -7 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. With two pivot columns and rows, Rank A=2 and dim Row A=2.

(b) (*8 points*) Find the bases for Col A and Row A.

Solution: Basis for Col A=$\left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} \right\}$ and Basis for Row A=$\left\{ \begin{bmatrix} 1 \\ -4 \\ 9 \\ -7 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -4 \\ 1 \end{bmatrix} \right\}$

Problem 2. (*4 points*) The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 3t - 6t^2$ relative to $\mathcal{B}$.

Solution: Using the standard basis $\{1, t, t^2\}$ for $\mathbb{P}_2$, we have the linear system whose augmented matrix is:

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \\ -1 & -1 & 1 & -6 \end{bmatrix}$$

Solve this linear system and we get:

$$[1 + 3t - 6t^2]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

Problem 3. (*6 points*) True or False. Provide a justification or a counter-example.

(a) $\mathbb{P}_2$ and $\mathbb{R}_3$ are isomorphic, i.e. there exists an isomorphism between the two spaces.

Solution: True, the change of coordinate basis matrix would be an isomorphism between the two spaces.

(b) Let W be a vector space spanned by the set $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. Then for all $\mathbf{x} \in W$, there exists a unique set of scalars c_1, c_2, c_3 such that $\mathbf{x} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + c_3\mathbf{b}_3$.

Solution: False, any $\mathcal{B}$ that is a linearly dependent set, then the constants wouldn't be unique.

(c) Let A be an $m \times n$ matrix. Rank A^T+Nullity A=n

Solution: True, Rank $A^T \equiv$ dim Row A=Rank A (number of pivots). And so by Rank Theorem the statement is true.