Math 53 Quiz 5
February 22, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). No calculators, please! Please read the instructions carefully, and explain your work.

Problem 1. (3 points) Prove that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy\, e^{x+y}}{x^2+y^2}$$

Taking the limit along the path $x=0$ gives a limiting value of

$$\lim_{y\to 0} \frac{(0)y e^{0+y}}{0^2+y^2} = \lim_{y\to 0} \frac{0}{y^2} = 0$$

while taking the limit along the path $x=y$ gives

$$\lim_{y\to 0} \frac{(y)y\, e^{(y)+y}}{(y)^2+y^2} = \lim_{y\to 0} \frac{y^2e^{2y}}{2y^2} = \lim_{y\to 0} \frac{e^{2y}}{2} = 1/2$$

Since the limits along different paths approaching $(0,0)$ are not the same, the limit doesn't exist.

Problem 2. (3 points) Determine the set of points at which the function

$$f(x,y) = \frac{e^x + e^y}{e^{x^2-y^2}-1}$$

is continuous.

Both the numerator and the denominator of f are continuous, so f is continuous whenever the denominator is nonzero. But we have

$$e^{x^2-y^2}-1=0 \iff e^{x^2-y^2}=1 \iff x^2-y^2=0$$

Thus f is continuous on its entire domain,

$$D=\{(x,y)\in \mathbb{R}^2 : x^2-y^2\neq 0\}$$

Problem 3. (4 points) Find an equation for the tangent plane to the surface

$$z = e^{x^2-y^2}$$

at the point $P = (1, 1, 1)$.

If we denote $f(x,y) = e^{x^2-y^2}$, then the tangent plane at the point P is given by

$$z-1 = f_x(1,1)(x-1) + f_y(1,1)(y-1)$$

Computing partial derivatives, we have

$$f_x(x,y) = 2x\,e^{x^2-y^2}, \quad f_x(1,1) = 2(1)e^{(1)^2-(1)^2} = 2$$

$$f_y(x,y) = -2y\,e^{x^2-y^2}, \quad f_y(1,1) = -2(1)e^{(1)^2-(1)^2} = -2$$

Thus the plane has equation

$$z-1 = 2(x-1) - 2(y-1).$$