Math 53 Quiz 4

February 15, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). Please read the instructions carefully, and explain your work. No calculators, please!

Problem 1. (3 points) Reduce the following equation to standard form, and classify the surface.

$$x^2 - y^2 - z^2 - 6x - 4y + 6 = 0$$

We first rearrange the left hand side of the equation and complete squares where appropriate.

$$x^2-y^2-z^2-6x-4y+6 = x^2-6x+9-9-y^2-4y-4+4-z^2+6$$
$$= (x-3)^2-9-(y+2)^2+4-z^2+6$$
$$= (x-3)^2-(y+2)^2-z^2+1$$

Rearranging the equation gives the standard form

$$-(x-3)^2+(y+2)^2+z^2 = 1$$

This represents a hyperboloid in one sheet.

Problem 2. (3 points) Find the unit tangent vector to the curve

$$\mathbf{r}(t) = \langle t^2 - 2t, 1 + 3t, t^3/3 + t^2/2\rangle$$

at the point $(0, 7, 14/3)$.

The value of t corresponding to the point $(0, 7, 14/3)$ can be found by equating the y coordinates: $1+3t=7 \to t=2$.

The tangent vector of $\vec{r}(t)$ is given by

$$\vec{r}'(t) = \langle 2t-2, 3, t^2+t\rangle$$

so a tangent vector at the desired point is $\vec{r}'(2) = \langle 2, 3, 6\rangle$.

To form a unit vector, we divide through by the magnitude $|\vec{r}'(2)| = \sqrt{2^2+3^2+6^2} = \sqrt{49} = 7$, to get a unit tangent vector

$$\vec{v} = \frac{\vec{r}'(2)}{|\vec{r}'(2)|} = \langle 2/7, 3/7, 6/7\rangle.$$

Problem 3. (4 points) Draw a contour map of the function $f(x, y) = 2y/(x^2 + y^2)$.

We will draw curves of the form $f(x,y) = c$ for a constant c.

$$2y/(x^2+y^2) = C$$
$$\rightarrow 2y = C(x^2+y^2)$$
$$\rightarrow x^2 + y^2 - 2y \cdot 1/c = 0$$
$$\rightarrow x^2+y^2-2y\cdot 1/c + 1/c^2 - 1/c^2 = 0$$
$$\rightarrow x^2 + (y - 1/c)^2 = 1/c^2.$$

Thus for $C \neq 0$, the contour is a circle of radius $|1/c|$ centered at $(0, 1/c)$. For $C = 0$, we just get the level curve $2y/(x^2+y^2) = 0 \rightarrow y = 0$ for $x \neq 0$. Together, the contour map looks like the following: