

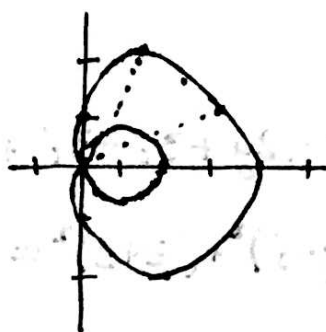
Math 53 Quiz 2
February 1, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). Please read the instructions carefully, and explain your work.

Problem 1. (3 points) Sketch the graph of the polar equation $r = 1 + 3 \cos \theta$, and give a parametric formula for the curve. We can sketch the graph using a table of values:

θ	$r(\theta)$	\approx
0	4	4
$\pi/6$	$1 + 3\sqrt{3}/2$	3.5
$\pi/4$	$1 + 3\sqrt{2}/2$	3.1
$\pi/3$	$1 + 3/2$	2.5
$\pi/2$	1	1
$2\pi/3$	$1 - 3/2$	-0.5
$3\pi/4$	$1 - 3\sqrt{2}/2$	-1.1
$5\pi/6$	$1 - 3\sqrt{3}/2$	-1.5
π	-2	-2

Using symmetry of the equation, we get the following graph:



A parametric formula is given by the equations
 $x = r \cos \theta$, $y = r \sin \theta$,
to get

$$x = (1 + 3 \cos \theta) \cos \theta$$

$$y = (1 + 3 \cos \theta) \sin \theta$$

Problem 2. (3 points) Find the length of the polar curve:

$$r = e^\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

Length of a polar curve is given by the formula

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta,$$

so in this case we get

$$\begin{aligned} L &= \int_{-\pi/2}^{\pi/2} \sqrt{e^{2\theta} + e^{2\theta}} d\theta = \sqrt{2} \int_{-\pi/2}^{\pi/2} e^\theta d\theta \\ &= \sqrt{2} (e^{\pi/2} - e^{-\pi/2}). \end{aligned}$$

Problem 3. (4 points) Find the distance between the spheres described by the following two equations:

$$x^2 + y^2 + z^2 = 2x, \quad x^2 + y^2 + z^2 = -4x + 12y - 4z - 40$$

We can complete the square several times to retrieve a canonical form for each of the spheres:

$$x^2 - 2x + 1 - 1 + y^2 + z^2 = 0$$

$$\rightarrow (x-1)^2 + y^2 + z^2 = 1$$

and

$$x^2 + 4x + 4 - 4 + y^2 - 12y + 36 - 36 + z^2 + 4z + 4 - 4 = -40$$

$$\rightarrow (x+2)^2 + (y-6)^2 + (z+2)^2 = 4$$

Thus the first sphere has center $(1, 0, 0)$ and radius 1, and the second sphere has center $(-2, 6, -2)$ and radius 2. The distance between the two center points is just

$$\sqrt{(1-(-2))^2 + (0-6)^2 + (0-(-2))^2} = \sqrt{9+36+4} = 7,$$

so the distance between the spheres is this distance minus the two radii, or $7-1-2=4$.