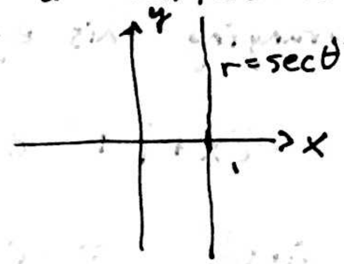
(a)

Math 53 Quiz 2
February 1, 2017

This quiz will be graded out of 10 points, with individual questions weighted as (indicated). Please read the instructions carefully, and explain your work.

Problem 1. (3 points) Sketch the graph of the polar equation $r = \sec\theta$, and give a Cartesian equation representing the same curve.

We know that $\sec\theta = \frac{1}{\cos\theta}$, so the polar equation may be rewritten as $r = 1/\cos\theta$. Rearranging using algebra, we see that for any point on the curve, we must have $x = r\cos\theta = 1$. In fact, $x = 1$ is a Cartesian equation for the curve, so the graph is given by the following:



Problem 2. (3 points) If $r_1 = 2 - \cos\theta$ and $r_2 = 1 + \cos\theta$, find the area of the region that is outside the curve described by r_1 and inside the curve described by r_2.

The two functions r_i are nonnegative and periodic mod 2π, so we need only find the angles θ in an interval of length 2π for which $r_1 \leq r_2$. Since the functions are continuous, the ranges of angles must be bounded by angles where $r_1 = r_2$:

$$2 - \cos\theta = 1 + \cos\theta \longrightarrow \cos\theta = 1/2 \longrightarrow \theta = \pm \pi/3 + 2\pi k.$$

So checking relative values, we see that in the interval $[-\pi, \pi]$, the range $\theta \in [-\pi/3, \pi/3]$ gives the angles we want that describe the region. Then the area is given by

$$\int_{-\pi/3}^{\pi/3} \frac{1}{2}(r_2^2 - r_1^2)\,d\theta = \int_{-\pi/3}^{\pi/3} \frac{1}{2}\left((1+2\cos\theta+\cos^2\theta)-(4-4\cos\theta+\cos^2\theta)\right)d\theta$$

$$= \int_{-\pi/3}^{\pi/3} (-3/2 + 3\cos\theta)\,d\theta = (-3/2\,\theta + 3\sin\theta)\Big|_{-\pi/3}^{\pi/3} = 3\sqrt{3} - \pi.$$

Problem 3. (4 points) Find an equation for the set of points in space that are equidistant from the point $A = (1, 1, 0)$ and the point $B = (0, 1, 1)$. What geometric object does this set represent?

A point $P = (x, y, z)$ is equidistant from A and B if

$$\sqrt{(x-1)^2 + (y-1)^2 + z^2} = \sqrt{x^2 + (y-1)^2 + (z-1)^2}$$

or equivalently if

$$(x-1)^2 + (y-1)^2 + z^2 = x^2 + (y-1)^2 + (z-1)^2.$$

Rearranging this equation, we get that

$$-2x + 1 + 2z - 1 = 0$$

or just $z - x = 0$. In particular this equation represents a plane in space.