Worksheet 11, Math 53
Line Integrals

Wednesday, November 7, 2012

1. If C is a smooth curve given by a vector function $\mathbf{r}(t)$, $a \le t \le b$, and $\mathbf{v}$ is a constant vector, show that
$$\int_C \mathbf{v} \cdot d\mathbf{r} = \mathbf{v} \cdot [\mathbf{r}(b) - \mathbf{r}(a)]$$

2. Determine whether or not $\mathbf{F}(x, y) = e^x \cos y\, \mathbf{i} + e^x \sin y\, \mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

3. Suppose you're asked to determine the curve that requires the least work for a force field $\mathbf{F}$ to move a particle from one point to another. You decide to check first whether $\mathbf{F}$ is conservative, and indeed it turns out that it is. How would you reply to the request?

4. Evaluate the line integral
$$\int_C z^2\, dx + x^2\, dy + y^2\, dz,$$
where C is the line segment from $(1, 0, 0)$ to $(4, 1, 2)$.

5. Evaluate the line integral
$$\int_C \mathbf{F} \cdot d\mathbf{r},$$
where $\mathbf{F}(x, y) = xy^2\, \mathbf{i} + x^2 y\, \mathbf{j}$, and C is given by the vector function $\mathbf{r}(t) = (t + \sin(\frac{1}{2}\pi t))\, \mathbf{i} + (t + \cos(\frac{1}{2}\pi t))\, \mathbf{j}$, with $0 \le t \le 1$.

6. Evaluate the line integral
$$\int_C \mathbf{F} \cdot d\mathbf{r},$$
where $\mathbf{F}(x, y, z) = \sin x\, \mathbf{i} + \cos y\, \mathbf{j} + xz\, \mathbf{k}$, and C is given by the vector function $\mathbf{r}(t) = t^3\, \mathbf{i} - t^2\, \mathbf{j} + t\, \mathbf{k}$, with $0 \le t \le 1$.