

## Worksheet 9, Math 53

### Double and Triple Integrals

Wednesday, October 24, 2012

1. Verify that the function

$$f(x, y) = \begin{cases} 4xy & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

is a joint density function.

If  $X$  and  $Y$  are random variables with joint density function  $f$ , find  $P(X \geq \frac{1}{2})$ ,  $P(X \geq \frac{1}{2}, Y \leq \frac{1}{2})$ , and the expected values of  $X$  and  $Y$ .

2. Find the center of mass of a lamina in the shape of an isosceles right triangle with equal sides of length  $a$  if the density at any point is proportional to the square of the distance from the vertex opposite the hypotenuse.
3. Find the surface area of the finite part of the paraboloid  $y = x^2 + z^2$  cut off by the plane  $y = 25$ .
4. Use a triple integral to find the volume of the solid enclosed by the paraboloids  $y = x^2 + z^2$  and  $y = 8 - x^2 - z^2$ .
5. Use cylindrical coordinates to find the volume of the solid that lies within both the cylinder  $x^2 + y^2 = 1$  and the sphere  $x^2 + y^2 + z^2 = 4$ .
6. Use spherical coordinates to find the volume of the part of the ball  $\rho \leq a$  that lies between the cones  $\phi = \pi/6$  and  $\phi = \pi/3$ .