

Worksheet 8, Math 53

Double Integrals

Wednesday, October 17, 2012

1. Find the average value of $f(x, y) = x^2y$ over the rectangle $R = [-1, 1] \times [0, 5]$.
2. Use symmetry to evaluate

$$\iint_R \frac{xy}{1+x^4} dA, R = [-1, 1] \times [0, 1].$$

3. In evaluating a double integral over a region D , a sum of iterated integrals was obtained as follows:

$$\iint_D f(x, y) dA = \int_0^1 \int_0^{2y} f(x, y) dx dy + \int_1^3 \int_0^{3-y} f(x, y) dx dy.$$

Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

4. Use geometry to evaluate the double integral

$$\iint_D \sqrt{R^2 - x^2 - y^2} dA,$$

where D is the disk with center at the origin and radius R .

5. Use polar coordinates to find the volume of the solid consisting of points contained inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.
6. Let D be the disk with center the origin and radius a . What is the average distance from points in D to the origin?