Worksheet 6, Math 53
Tangent Planes and Gradient Vectors

Wednesday, October 3, 2012

1. Let S be the surface represented by the equation $xy + yz + zx = 5$, and let $P(1, 2, 1)$ be a point on S. Find an equation of the tangent plane of S at P.

2. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S, but you know that the curves

$$\mathbf{r}_1(t) = \left\langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \right\rangle$$
$$\mathbf{r}_2(u) = \left\langle 1 + u^2, 2u^3 - 1, 2u + 1 \right\rangle$$

   both lie on S. Find an equation of the tangent plane of S at P.

3. Show that the sum of the x-, y-, and z-intercepts of any tangent plane to the surface $\sqrt{x}+\sqrt{y}+\sqrt{z} = \sqrt{c}$ is a constant.

4. Find the maximum rate of change of $f(x, y) = \sin(xy)$ at the point $(x, y) = (1, 0)$, and determine the direction in which this rate of change is achieved.

5. Two surfaces are called *orthogonal* at a point of intersection if their normal lines are perpendicular at that point.

   (a) Show that surfaces with equations $F(x, y, z) = 0$ and $G(x, y, z) = 0$ are orthogonal at a point P where $\nabla F \neq \mathbf{0}$ and $\nabla G \neq \mathbf{0}$ if and only if

$$F_x G_x + F_y G_y + F_z G_z = 0 \quad \text{at } P$$

   (b) Use this fact to show that the surfaces $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = r^2$ are orthogonal at every point of intersection.