

# Worksheet 3, Math 53

## Vectors and Vector Products

Wednesday, September 12, 2012

1. Suppose that  $\mathbf{a} \neq \mathbf{0}$ .
  - (a) If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?
  - (b) If  $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?
  - (c) If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$  and  $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?
2. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
3. Let  $r > 1$ , and consider all of the points which are  $r$  times the distance from  $A(-1, 0, 0)$  as from  $B(1, 0, 0)$ . Show that this set of points is a sphere, and find the center and radius of this sphere in terms of  $r$ .
4. Let  $\mathbf{u}$  and  $\mathbf{v}$  be vectors in  $\mathbf{R}^3$ . Show that  $\mathbf{u}$  and  $\mathbf{v}$  are perpendicular if and only if  $|\mathbf{u} + \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2$ . What is the name of this famous theorem?
5. Suppose that one side of a triangle forms the diameter of a circle and the vertex opposite this side lies on this circle. Use the dot product to prove that this is a right triangle.