Worksheet 6, Math 1B

Comparison, Ratio, and Root Tests; Alternating Series

Monday, February 27, 2012

1. Determine whether the series converges absolutely, converges conditionally, or diverges:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^k$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$

   (h) $\displaystyle 1 - \frac{1\cdot 3}{3!} + \frac{1\cdot 3\cdot 5}{5!} - \frac{1\cdot 3\cdot 5\cdot 7}{7!} + \cdots + (-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{(2n-1)!} + \cdots$

2. If $\sum a_n$ is a convergent series with positive terms, is it true that $\sum \sin(a_n)$ is also convergent?

3. If $\sum a_n$ and $\sum b_n$ are both convergent series with positive terms, is it true that $\sum a_n b_n$ is also convergent?

4. How many terms of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$$

   do we need to add in order to find the sum up to an error of 10^{-4}?