Worksheet 10, Math 1B
Separable Differential Equations

Monday, April 2, 2012

1. Find the solution of the differential equation $\frac{\mathrm{d}P}{\mathrm{d}t} = \sqrt{Pt}$, that satisfies the initial condition $P(1) = 2$.
2. Find the solution of the differential equation $y' \tan x = a + y$, that satisfies the initial condition $y(\pi/3) = a$, $0 < x < \pi/2$.
3. Solve the differential equation $xy' = y + xe^{y/x}$ by making the change of variable $v = y/x$.
4. Sketch the direction field of the differential equation $y' = y + xy$, and use it to sketch a solution corve that passes through the point $(0, 1)$.
5. According to Newton's Law of Universal Gravitation, the gravitational force on an object of mass m that has been projected vertically upward from the earth's surface is

$$F = \frac{mgR^2}{(x+R)^2}$$

where $x = x(t)$ is the object's distance above the surface at time t, R is the earth's radius, and g is the constant of acceleration due to gravity. Also, by Newton's Second Law, $F = ma = m(dv/dt)$, and so

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{mgR^2}{(x+R)^2}$$

(a) Suppose a rocket is fired vertically upward with an initial velocity v_0. Let h be the maximum height above the surface reached by the object. Show that

$$v_0 = \sqrt{\frac{2gRh}{R+h}}$$

[*Hint:* By the Chain Rule, $m(dv/dt) = mv(dv/dx)$.]

(b) Calculate $v_e = \lim_{h\to\infty} v_0$. This limit is called the *escape velocity* for the earth.

(c) Use $R = 3960$ miles and $g = 32\mathrm{ft/s}^2$ to calculate v_e in feet per second.