Quiz 5 Solutions, Math 1B, Section 309

Friday, April 13, 2012

Name:

Student ID#:

Please place personal items under your seat.
No use of notes, texts, calculators, or fellow students is allowed.
Show all of your work in order to receive full credit.

1. Classify the following differential equations by filling in the following table.

   For the first column, fill in a number, and for the remaining columns, fill in “Yes”, “No”, or “N/A” to indicate that the notion is not applicable to the given equation.

   Out of 5 points, -1/2 point per incorrect entry, rounded down.

   Solution

	Order	Separable	Linear	Homogeneous
$yy' - y' - x = 0$	1	Yes	No	N/A or No
$y' - xy = 0$	1	Yes	Yes	Yes
$y''' - 3xy' + x^2 = 0$	3	N/A or No	Yes	No
$y''/x + y' - y = 0$	2	N/A or No	Yes	Yes

2. Find the general solution to the following differential equation:

$$y' = \frac{\sqrt{x}}{e^y}$$

   Solution

   While not linear, this first-order differential equation is separable, and so we may write $e^y dy = \sqrt{x}dx$ and integrate to get

$$e^y = \frac{2}{3}x^{3/2} + C.$$

Taking a logarithm, this gives us

$$y = \ln\left(\frac{2}{3}x^{3/2} + C\right).$$

3. Find the solution to the following initial value problem.

$$y'' + 2y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 1$$

Solution

We first find the general solution of the equation, and then use the initial values to find the appropriate specific solution. Since this is a second order linear differential equation which is homogeneous and has constant coefficients, we can use the characteristic equation method.

The characteristic equation is given by

$$r^2 + 2r + 2 = 0,$$

and so using the quadratic formula, we see that it has solutions

$$r = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)} = -1 \pm i,$$

and this corresponds to a general solution of

$$y = C_1 e^{-x}\cos(x) + C_2 e^{-x}\sin(x).$$

Calculating the derivative of this, we find

$$\begin{aligned} y' &= -C_1 e^{-x}\sin(x) - C_1 e^{-x}\cos(x) + C_2 e^{-x}\cos(x) - C_2 e^{-x}\sin(x) \\ &= (C_2 - C_1)e^{-x}\cos(x) + (-C_1 - C_2)e^{-x}\sin(x). \end{aligned}$$

Then plugging in the initial conditions gives us

$$\begin{cases} y(0) = 2 = C_1 \\ y'(0) = 1 = C_2 - C_1 \end{cases},$$

which has solution $C_1 = 2$, $C_2 = 3$. Thus the solution to the initial value problem is

$$y = 2e^{-x}\cos(x) + 3e^{-x}\sin(x).$$