Quiz 3 Solutions, Math 1B, Section 309

Friday, March 2, 2012

Name:

Student ID#:

Please place personal items under your seat.
No use of notes, texts, calculators, or fellow students is allowed.
Show all of your work in order to receive full credit.

Please pick your favorite **four** out of the following **five** problems to solve. Circle the numbers for the problems that you choose.

Do the following series converge or diverge? Justify your answer.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

   Solution

   This series has alternating terms which have absolute value $1/\sqrt{n+1}$. In particular, this sequence is decreasing (since the square-root function is increasing), and has limit zero. Thus by the alternating series test, the series converges.

2. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$

   Solution

   Since the series has terms which involve a function of n raised to a power which is a function of n, we try the root test.

$$\lim_{n\to\infty} \left| \frac{n}{(\ln n)^n} \right|^{1/n} = \lim_{n\to\infty} \frac{n^{1/n}}{\ln n} = 0.$$

   The last equality follows because the limit of the top of the fraction is 1, while the bottom increases to positive infinity. Thus since the limit is less than 1, the root test tells us that the series is absolutely convergent, and therefore convergent.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(n-2)(n+1)}$

Solution

The terms of the series have limit given by

$$\lim_{n\to\infty} \frac{n^2}{n^2-n-2} = \lim_{n\to\infty} \frac{1}{1-\frac{1}{n}-\frac{2}{n^2}} = 1.$$

Therefore, since the limit is not zero, the Test for Divergence gives us that the series is divergent.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

Solution

The series has all positive terms, so we may apply the limit comparison test with $b_n = 1/n$. Then

$$\lim_{n\to\infty} \frac{n^2/(n^3+1)}{1/n} = \lim_{n\to\infty} \frac{n^3}{n^3+1} = 1,$$

and so by the limit comparison test, since $\sum_{n=0}^{\infty} 1/n$ is divergent (the harmonic series), so is the series in question.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot n!}{20^n}$

Solution

Since the terms of the series involve simple exponential and factorial expressions, we use the ratio test.

$$\lim_{n\to\infty} \left| \left(\frac{2\cdot(n+1)!}{20^{n+1}}\right) \Big/ \left(\frac{2\cdot n!}{20^n}\right) \right| = \lim_{n\to\infty} \frac{n}{20} = \infty$$

Thus, since the limit is greater than 1, the series diverges.