Name:

Math 110, Section 105, Quiz 13
Wednesday, November 29, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ In a finite-dimensional real inner product space, any normal operator is orthogonally diagonalizable.

b. ________ A self-adjoint operator in a finite-dimensional real inner product space has a symmetric matrix with respect to any orthonormal basis.

c. ________ If T is a linear operator on a finite-dimensional inner product space and c is a scalar, then $(cT)^* = c(T^*)$.

Solution. F T F

Exercise. If $T : \mathbb{C}^2 \to \mathbb{C}^2$ is the linear operator given by

$$T(z_1, z_2) = \Big(z_1 \, + \, (1+i)z_2, \; (2-i)z_1 \, + \, 3z_2\Big)$$

find the adjoint operator T^*. Determine whether T^* is normal.

Solution. The linear operator T can be represented in the standard basis on $\mathbb{C}^2$ by the matrix

$$[T]_E = \begin{pmatrix} 1 & 1+i \\ 2-i & 3 \end{pmatrix}$$

In particular, since the standard basis E is orthonormal, we know that $[T^*]_E = [T]_E^*$. Thus

$$[T^*]_E = \begin{pmatrix} 1 & 2+i \\ 1-i & 3 \end{pmatrix}$$

As a map on $\mathbb{C}^2$, we have

$$T(z_1, z_2) = \Big(z_1 \, + \, (2+i)z_2, \; (1-i)z_1 \, + \, 3z_2\Big)$$

T^* is normal when $T^*(T^*)^* = (T^*)^*T^*$, which is equivalent to the condition $TT^* = T^*T$. Thus we can check whether $[T]_E[T^*]_E \neq [T^*]_E[T]_E$. By a direct computation, we can see that $[T]_E[T^*]_E \neq [T^*]_E[T]_E$, so we know that T^* is not a normal operator.