Name:

Math 110, Section 103, Quiz 13
Wednesday, November 29, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ In a finite-dimensional complex inner product space, any self-adjoint operator is orthogonally diagonalizable.

b. ________ If T is a normal operator on a finite-dimensional inner product space V, then $\|T(x)\| = \|T^*(x)\|$ for any $x \in V$.

c. ________ If T and U are linear operators on a finite-dimensional inner product space, then $(TU)^* = U^*T^*$.

Solution. T T T

✷

Exercise. Prove that the following 2×2 complex matrix A is normal, but not Hermitian. Is A orthogonally diagonalizable?

$$A = \begin{pmatrix} i & -2 \\ 2 & i \end{pmatrix}$$

Solution. The conjugate transpose of A is given by

$$A^* = \begin{pmatrix} -i & 2 \\ -2 & -i \end{pmatrix}$$

In particular, $A^* \neq A$, so A is not Hermitian. However, we can show that $AA^* = A^*A$ in a number of ways. One way is by directly computing these matrix products. Another is to notice that for this matrix A, we have $A^* = -A$, so in particular $A^*A = (-A)A = -A^2 = A(-A) = AA^*$. Thus A is normal.

For the last part, note that in a complex vector space, an operator is orthogonally diagonalizable if and only if it is normal. Since A is normal, it is orthogonally diagonalizable.