Name:

Math 110, Section 105, Quiz 12
Wednesday, November 15, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ For any finite dimensional real or complex vector space V, there always exists some inner product defined on V.

b. ________ The Cauchy-Bunyakovsky-Schwarz inequality states that for two vectors x, y in an inner product space, $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$.

c. ________ An inner product on an arbitrary vector space is linear in the second component.

Solution. T T F

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Exercise. Use the fact that $\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx$ is an inner product on the vector space $C([0,1])$ of continuous real-valued functions on the interval $[0,1]$ to prove that

$$\left|\int_0^1 x^4 e^{2x}\,dx\right| \leq \frac{\sqrt{e^4-1}}{6}$$

Solution. The following application of the Cauchy-Bunyakovsky-Schwarz inequality allows us to conclude the upper bound in question.

$$\left|\int_0^1 x^4 e^{2x}\,dx\right| = |\langle x^4, e^{2x}\rangle| \leq \|x^4\| \cdot \|e^{2x}\|$$
$$= \sqrt{\int_0^1 x^8\,dx} \cdot \sqrt{\int_0^1 e^{4x}\,dx} = \sqrt{1/9} \cdot \sqrt{(e^4-1)/4} = \frac{\sqrt{e^4-1}}{6}$$

Interesting to note is that the bound obtained in this case is fairly close to the actual value of the integral. The integral has precise value $(e^2-3)/4 \approx 1.10$, while the upper bound is $\sqrt{e^4-1}/6 \approx 1.22$.