Name:

Math 110, Section 101, Quiz 12

Wednesday, November 15, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ For an $n \times n$ complex matrix A, the adjoint of the adjoint, $(A^*)^*$, is equal to A.

b. ________ An inner product is a scalar-valued function on the set of vectors in a vector space.

c. ________ If x is a vector in an inner product space and c is a scalar, then $\|cx\| = c \cdot \|x\|$.

Solution. T F F

Exercise. Prove that the map on pairs of 2×2 real matrices given by

$$\langle A, A' \rangle = \left\langle \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \right\rangle = aa' + cb'$$

is *not* an inner product on $M_{2\times 2}(\mathbb{R})$.

Solution. The map fails to be an inner product on $M_{2\times 2}(\mathbb{R})$ for a number of reasons. As one example, let A be the matrix

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Then $\langle A, A \rangle = 0 \cdot 0 + 0 \cdot 0 = 0$, but A is not the zero matrix. This is already enough, but in addition, the map is not symmetric. If A and B are the matrices

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

then $\langle A, B \rangle = 1 \cdot 1 + 1 \cdot 1 = 2$, while $\langle B, A \rangle = 1 \cdot 1 + 0 \cdot 0 = 1$.