Name:

Math 110, Section 105, Quiz 11
Wednesday, November 8, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ The Jordan canonical form of a generalized eigenspace is a single Jordan block when its corresponding dot diagram is a single column.

b. ________ If T is a linear operator on a finite-dimensional vector space V whose characteristic polynomial splits, then V is the direct sum of the generalized eigenspaces of T.

c. ________ The number of columns of the dot diagram corresponding to an eigenvalue λ is equal to the dimension of the generalized eigenspace K_λ.

Solution. T T F

✷

Exercise. Let A be a complex matrix with an eigenvalue $\lambda = 3$, such that the dot diagram of the generalized eigenspace K_3 is given by

$$\begin{array}{ccccc} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \\ \bullet & \bullet & & & \\ \bullet & \bullet & & & \end{array}$$

What are the algebraic and geometric multiplicities of $\lambda = 3$? Give an expression for a matrix W such that $K_3 = N(W)$.

Solution. The algebraic multiplicity of $\lambda = 3$ is given by the number of dots in the dot diagram of K_3, and the geometric multiplicity is given by the number of columns. Thus we have

$$m_a(3) = 13, \qquad m_g(3) = 5$$

In general, the generalized eigenspace is given by $N(U^k)$, where $U = A - \lambda I$ and k is the stabilizing constant, or the smallest k such that $\dim(N(U^k)) = \dim(N(U^{k+1}))$. In particular, in terms of the dot diagram for K_λ, the stabilizing constant is equal to the number of rows of the diagram. Thus we have

$$K_3 = N\big((A - 3I)^4\big)$$

Note that in general, any exponent *larger* than the stabilizing constant will also work, so a safe bet in any scenario will be the algebraic multiplicity since the number of rows is certainly at most the number of dots in the diagram. Thus one could also write $K_3 = N\big((A - 3I)^{13}\big)$ in this case.