Name:

Math 110, Section 101, Quiz 11
Wednesday, November 8, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ Every linear operator with the characteristic polynomial $p(t) = (-1)^n(t-\lambda)^n$ has the same Jordan canonical form.

b. ________ A cycle of generalized eigenvectors is linearly independent.

c. ________ Suppose that the longest row of the dot diagram for a generalized eigenspace K_λ has 5 dots. Then there is a cycle of generalized eigenvectors for λ of length 5.

Solution. F T F

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Exercise. Let A and v be the 3×3 matrix and vector given below. Show that v is a generalized eigenvector of A with eigenvalue 2 by computing its corresponding cycle of generalized eigenvectors. Use what you find to give the Jordan canonical form of A. (Hint: Apply the Halloween operator.)

$$A = \begin{pmatrix} -1 & 12 & -3 \\ 0 & 4 & 1 \\ 1 & -4 & 3 \end{pmatrix}, \qquad v = \begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$$

Solution. The Halloween operator $U = A - 2I$ is given by

$$U = \begin{pmatrix} -3 & 12 & -3 \\ 0 & 2 & 1 \\ 1 & -4 & 1 \end{pmatrix}$$

To compute the cycle generated by v, we compute the vectors $U^k v$ for $k = 1, 2, 3, \ldots$ until we find a k such that $U^k v = 0$. The existence of this k shows that v is a generalized eigenvector. We have

$$U(v) = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}, \qquad U^2(v) = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix}, \qquad U^3(v) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Thus v is a generalized eigenvector. In particular, the cycle of v consists of 3 linearly independent vectors in K_2, which implies that the generalized eigenspace K_2 is equal to the underlying vector space. Further, this implies that the dot diagram of K_2 is just a column with 3 dots, which shows that the Jordan canonical form of A is

$$J = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$