Name:

Math 110, Section 103, Quiz 9
Wednesday, October 25, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ A linear operator T on an n-dimensional vector space is diagonalizable iff its characteristic polynomial splits, and the geometric multiplicity of each eigenvalue λ is equal to $\dim(\ker(T - \lambda I))$.

b. ________ If T is a linear operator, then any T-cyclic subspace of an eigenspace has dimension 1.

c. ________ If T is a linear operator and W is a T-invariant subspace, then the characteristic polynomial of T_W (the restriction of T to the subspace W) divides the characteristic polynomial of T.

Solution. F T T

✷

Exercise. Let $V = M_{n\times n}(\mathbb{R})$ for some $n \geq 2$, and let W be the subspace of skew-symmetric real matrices. If $T : A \mapsto A^t$ is the transpose operator, prove that W is T-invariant. When is W also T-cyclic? Explain.

Solution. If A is skew-symmetric, we need to show that $T(A)$ is skew-symmetric as well. We have $T(A) = A^t = -A$, so $(T(A))^t = (-A)^t = -(A^t) = -T(A)$. Thus $T(A)$ is also skew-symmetric, and we see that W is T-invariant.

Notice now that because W is the eigenspace of T corresponding to the eigenvalue $\lambda = -1$. In particular, this means that any T-cyclic subspace of W has dimension 1, since $A, T(A)$ is linearly dependent for any skew-symmetric matrix. Thus W cannot be T-cyclic if $\dim(W) > 1$, which is the case if $n > 2$. For $n = 2$, W has dimension 1, and in particular is the T-cyclic subspace generated by the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Thus W is T-cyclic if $n = 2$, and is not T-cyclic if $n > 2$.