Name:

Math 110, Section 105, Quiz 8
Wednesday, October 18, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ Any two eigenvectors are linearly independent.

b. ________ There exists a square matrix with no eigenvectors.

c. ________ If λ is an eigenvalue of a linear operator T, then each vector in its eigenspace E_λ is an eigenvector of T.

Solution. F T F

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Exercise. For the matrix

$$A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$$

find a diagonal matrix D and a change of coordinates matrix Q such that $Q^{-1}AQ = D$.

Solution. We compute the characteristic polynomial

$$\mathrm{char}_A(\lambda) = \det(A - \lambda I) = (\lambda - 1)(\lambda - 2) - 12 = \lambda^2 - 3\lambda - 10 = (\lambda - 5)(\lambda + 2)$$

Thus the eigenvalues of A are $\lambda = 5$ and $\lambda = -2$. The corresponding matrices $A - \lambda I$ and row-reduced forms are

$$\begin{pmatrix} -4 & 4 \\ 3 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 4/3 \\ 0 & 0 \end{pmatrix}$$

For $\lambda = 5$, the null space is described by $x = y$, so it is spanned by the vector $(1, 1)$. For $\lambda = 5$, the null space is described by $x = -4/3y$, so it is spanned by the vector $(-4, 3)$. Together, these vectors form an eigenbasis $\beta = \{(1, 1), (-4, 3)\}$ for A. The desired matrix Q is formed by using the vectors in β as columns, and the diagonal matrix D is formed by placing the eigenvalues corresponding to the vectors of β in the diagonal entries. Thus we obtain

$$Q = \begin{pmatrix} 1 & -4 \\ 1 & 3 \end{pmatrix}, \qquad D = \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$$