Name:

Math 110, Section 103, Quiz 8
Wednesday, October 18, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ A diagonalizable linear operator on a nonzero vector space has at least one eigenvalue.

b. ________ If a linear operator T has a characteristic polynomial that splits over its underlying field, then T is diagonalizable.

c. ________ Similar matrices always have the same eigenvalues.

Solution. T F T

✷

Exercise. If $A \in M_{2\times 2}(\mathbb{C})$ is the matrix given by

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

find the eigenvalues of A and a basis of eigenvectors. Is A diagonalizable over $\mathbb{R}$?

Solution. We compute the characteristic polynomial

$$\mathrm{char}_A(\lambda) = \det(A - \lambda I) = \lambda^2 + 1 = (\lambda - i)(\lambda + i)$$

Thus the eigenvalues of A are $\lambda = i$ and $\lambda = -i$. The corresponding matrices $A - \lambda I$ and row-reduced forms are

$$\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \sim \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \sim \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix}$$

For $\lambda = i$, the null space is thus given by the relation $x = iy$, so it is spanned by $(i, 1)$. For $\lambda = -i$, the null space is given by the relation $x = -iy$, so it is spanned by $(-i, 1)$. Thus a basis of eigenvalues is given by the combination of these two vectors, $\beta = \{(i, 1), (-i, 1)\}$.

Finally, A is not diagonalizable over $\mathbb{R}$. This is because the characteristic polynomial of A has no roots in $\mathbb{R}$, and so A has no eigenvalues over this field, hence does not admit a basis of eigenvectors.