Name:

Math 110, Section 101, Quiz 8
Wednesday, October 18, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ If λ_1 and λ_2 are distinct eigenvalues of a linear operator T with corresponding eigenspaces E_{λ_1} and E_{λ_2}, then $E_{\lambda_1} \cap E_{\lambda_2} = \{0\}$.

b. ________ A real matrix may have exactly two eigenvectors.

c. ________ The sum of two linearly independent eigenvectors is never an eigenvector.

Solution. T F F

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Exercise. Find a basis of eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

Solution. We compute the characteristic polynomial

$$\mathrm{char}_A(\lambda) = \det(A - \lambda I) = (1-\lambda)(2-\lambda)(1-\lambda) - (2-\lambda) = \lambda(\lambda-2)^2$$

Thus we have two eigenvalues $\lambda = 0$ and $\lambda = 2$. The corresponding matrices $A - \lambda I$ and row-reduced forms are

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For $\lambda = 0$, we compute the null space by $x_1 = x_3$ and $x_2 = 0$, so the space is spanned by $(1, 0, 1)$. For $\lambda = 2$, the null space is described by $x_1 = x_2 - x_3$, so it is spanned by $(1, 1, 0)$ and $(1, 0, -1)$. Thus an eigenbasis for A is given by the union of these spanning sets:

$$\beta = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$