Math 110, Section 105, Quiz 6
Wednesday, October 4, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ The system of linear equations $Ax = b$ has a solution if and only if b is in the range of the left multiplication operator L_A.

b. ________ Any homogeneous linear system of equations has at least one solution.

c. ________ The inverse of an $n \times n$ elementary matrix is an elementary matrix.

Solution. T T T

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Exercise. Write the following linear system of equations as a matrix equation $Ax = b$, compute A^{-1}, and use it to solve the system.

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 - 2x_2 + x_3 &= 3 \end{aligned}$$

Solution. The matrix A and the vector b in the matrix equation can be read from the coefficients of the system:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & -2 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

Then A^{-1} can be computed by forming an augmented matrix of A with the 3×3 identity matrix and then row reducing A to the identity matrix.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & -4 & -1 & -2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 & 2 & -1 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -9 & -6 & 4 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 3 & 2 & -1 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2/3 & -4/9 & -1/9 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & 1/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 1/9 & -2/9 \\ 0 & 0 & 1 & 2/3 & -4/9 & -1/9 \end{pmatrix}$$

Thus we compute

$$A^{-1} = \begin{pmatrix} 0 & 1/3 & 1/3 \\ 1/3 & 1/9 & -2/9 \\ 2/3 & -4/9 & -1/9 \end{pmatrix}, \quad x = A^{-1}b = \begin{pmatrix} 4/3 \\ 1/9 \\ 5/9 \end{pmatrix}$$