Math 110, Section 103, Quiz 6
Wednesday, October 4, 2017

This quiz will be graded out of 15 points; the True/False question is worth 3 points, and the exercise is worth 12 points. Please read the instructions carefully, and explain your work.

True or False. Mark the following statements as either true or false, or leave a blank if you don't know. A correct answer is worth +1 point, a blank is worth 0 points, and an incorrect answer is worth -1 points, so be smart about guessing!

a. ________ An inhomogeneous linear system of equations may have the zero vector as a solution.

b. ________ Any invertible $n \times n$ matrix A satisfies $EA = I_n$ for some elementary matrix E.

c. ________ The solution set of a system of linear equations in n unknowns over a field F is always a subspace of F^n.

Solution. F F F

✷

Exercise. Write the following invertible 2×2 matrix as a product of elementary matrices.

$$A = \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}$$

Solution. Each elementary step in the process of row reducing A to the 2×2 identity matrix is equivalent to multiplying on the left by an elementary matrix. Thus the row reduction

$$\begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

corresponds with left multiplication by the elementary matrices

$$E_1 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}, \quad E_4 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

This gives us the product representation

$$I_2 = E_4 E_3 E_2 E_1 A$$

so we get that

$$A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}$$

or

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$